Econ 12200
Section 04

Lecture 16
October 5, 2016

1. Suppose the market for chocolate bars is characterized by the following equations: $P = 5 - \frac{1}{20}Q^D$ and $P = \frac{1}{20}Q^S$.

   (a) Graph the market for chocolate bars.

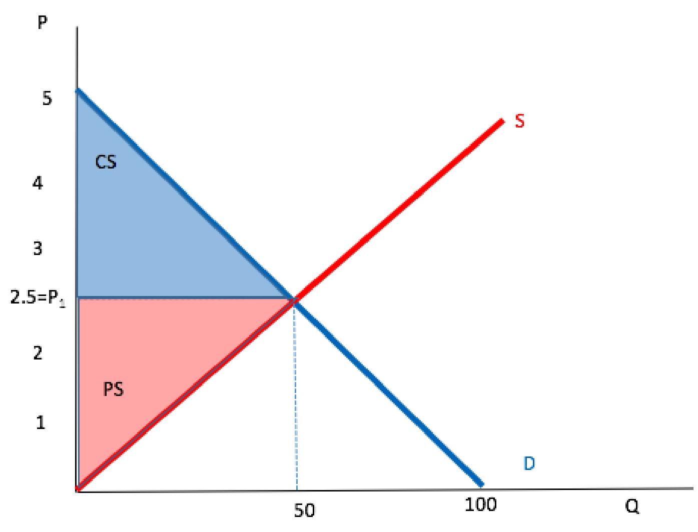
   (b) Calculate the consumer and producer surplus in equilibrium.

   (c) What is the total surplus in equilibrium?

   These equations already have P alone on the left hand side of the equation. Therefore, the constant is the intercept and the $\frac{1}{20}$ is the slope.

   First graph the demand curve, set $Q^D = 0$ to find that the y-intercept is 5. Then set $P = 0$ to find that the x-intercept is 100. Connect your two endpoints.

   Second, graph the supply curve, set $Q^S = 0$ to find that the y-intercept is 0. Then plug in the max quantity demanded $Q^D = 100$ to find that at $Q = 100$, $Q^S = 5$. Connect your dots.



   To solve for consumer and producer surplus in equilibrium we first need to solve for the equilibrium quantity and price.

   Set the two equations equal to one another.

   $5 - \frac{1}{20}Q^D = \frac{1}{20}Q^S$
   Set $Q^D = Q^S = Q$
   $5 - \frac{1}{20}Q = \frac{1}{20}Q$
   Multiply by 20 to eliminate the fraction

$100 - Q = Q$
Combine the Qs on one side
$100 = 2Q$
Divide by 2
$50 = Q$
Next plug in the equilibrium Q into one of the above equations to get price.
$P = \frac{1}{20}Q$
$P = \frac{1}{20} * 50$
$P = \frac{50}{20}$
$P = \frac{5}{2}$
$P = 2.5$

Consumer surplus is the area above the price (2.5) and below demand.

$CS = \frac{1}{2} * (5 - 2.5) * 50$
$CS = 0.5 * 2.5 * 50$
$CS = 62.5$

Producer surplus is the area below the price (2.5) and above supply.

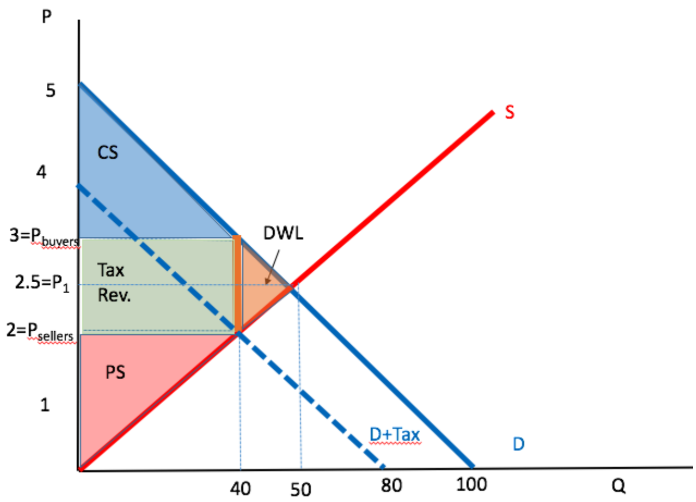
$PS = \frac{1}{2} * (2.5 - 0) * 50$
$PS = 0.5 * 2.5 * 50$
$PS = 62.5$

Total Surplus is the sum of producer surplus and consumer surplus.

$TS = PS + CS$
$TS = 62.5 + 62.5 = 125$

(d) Suppose we place a $1 tax on buyers chocolate bars. Solve for the following:

i. The price sellers receive.
ii. The price buyers pay.
iii. The new equilibrium quantity.
iv. The new consumer surplus.
v. The new producer surplus.
vi. The tax revenue.
vii. The DWL.



The tax changes the demand equation such that we now have $P + T = 5 - \frac{1}{20}Q^D$
We still want to get P alone so subtract the T to the other side of the equation.
$P = 5 - T - \frac{1}{20}Q^D$
Recall that $T = 1$
$P = 5 - 1 - \frac{1}{20}Q^D$
$P = 4 - \frac{1}{20}Q^D$

Now we have P alone on one side of the equation. Therefore, the constant (4) is the intercept and the $\frac{1}{20}$ is the slope.

First graph the demand curve, set $Q^D = 0$ to find that the y-intercept is 4. Then set $P = 0$ to find that the x-intercept is 80. Connect your two endpoints.

We're first going to solve for the new quantity. Set the new demand curve equal to the supply curve.

$P = 4 - \frac{1}{20}Q^D = \frac{1}{20}Q^S$
Set $Q^D = Q^S = Q$
$P = 4 - \frac{1}{20}Q = \frac{1}{20}Q$
Multiply by 20 to eliminate the fraction
$80 - Q = Q$
Combine the Qs on one side
$80 = 2Q$
Divide by 2
$40 = Q$
The new quantity with the tax is 40. Next we solve for the price sellers receive by plugging the new Q into the supply equation.
$P_{sell} = \frac{1}{20}Q$
$P = \frac{1}{20} * 40$
$P = \frac{40}{20}$
$P_{sell} = 2$

Next to get the price buyers pay, we can plug the new Q into the old demand equation.
$P_{buy} = 5 - \frac{1}{20}Q$
$P_{buy} = 5 - \frac{1}{20} * 40$
$P_{buy} = 5 - 2$
$P_{buy} = 3$

We could also have solved for the price buyers pay by simply adding the tax to the price sellers receive because the two prices will always differ by the tax.

$P_{buy} = P_{sell} + Tax$
$P_{buy} = 2 + 1$
$P_{buy} = 3$

To solve for the new consumer surplus we take the area below the original demand curve and above the price buyers pay.

$CS = \frac{1}{2} * (5 - p_{buy}) * Q_2$
$CS = \frac{1}{2} * (5 - 3) * 40$
$CS = \frac{1}{2} * 2 * 40$
$CS = 40$
To solve for the new producer surplus we take the area below the price sellers receive and above the supply curve.

$PS = \frac{1}{2} * (P_{sell} - 0) * Q_2$
$PS = \frac{1}{2} * (2 - 0) * 40$
$PS = 40$

Next we need to solve for tax revenue. Tax revenue is the tax (\$1) times the quantity sold with the tax (Q_2).

$\text{Tax Rev.} = T * Q_2$
$\text{Tax Rev.} = 1 * 40$
$\text{Tax Rev.} = 40$

Total Surplus is the sum of producer surplus, consumer surplus, and tax revenue.

$TS = PS + CS + \text{ Tax Rev.}$
$TS = 40 + 40 + 40 = 120$

To solve for the DWL triangle we have:

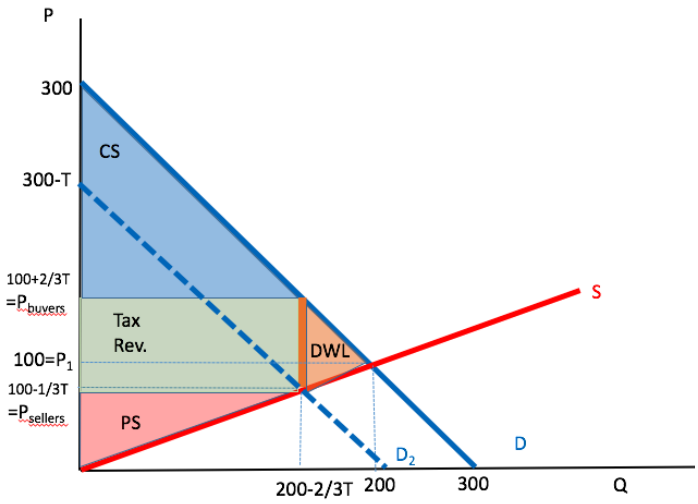
$DWL = \frac{1}{2} * (tax) * (Q_1 - Q_2)$
$DWL = \frac{1}{2} * (1) * (50 - 40)$
$DWL = \frac{1}{2} * (1) * (10)$
$DWL = 5$

Notice that the change in total surplus matches the DWL that we solved for.

2. Suppose an market is characterized by the following:
$Q^S = 2P$ and $Q^D = 300 - P$.

   (a) Graph the market. What is the equilibrium price and quantity? First, we should rearrange our equations so that P is alone on the left hand side. For demand we have: $P = 300 - Q^D$
   Then set $Q^D = 0$ to see that the y-axis intercept is 300. Then set $P = 0$ to see that the x-axis intercept is 300.

   For supply we have: $P = \frac{1}{2}Q^S$ Then set $Q^S = 0$ to see that the y-axis intercept is 0. Then set $Q^S = 300$ to see that at the max Q^D, the price is 150.



   (b) Suppose the government decides to tax buyers such that demand is now characterized by $Q^D = 300 - (P + T)$. Graph the demand curve with the tax.

   Again we want to rearrange our equation so that we have P alone on the left hand side.
   $Q^D = 300 - (P + T)$ $P = 300 - T - Q^D$ Then set $Q^D = 0$ to see that the y-axis intercept is $300 - T$. Then set $P = 0$ to see that the x-axis intercept is $300 - T$.

   (c) Solve for the price sellers receive in terms of the tax (T).

   To solve for the price sellers receive we need to set the new demand equation equal to the supply equation.
   $2P = 300 - (P + T)$
   $3P = 300 - T$
   $P_{sell} = 100 - \frac{1}{3}T$

   (d) Solve for the price buyers pay in terms of the tax.

   To solve for the price buyers pay, we can add the tax (T) to the price sellers receive.

$P_{buy} = P_{sell} + T$
$P_{buy} = (100 - \frac{1}{3}T) + T$
$P_{buy} = 100 + \frac{2}{3}T$

(e) Solve for the new equilibrium quantity in terms of the tax.

To solve for the new equilibrium quantity we plug P_{sell} into the supply equation.

$Q_2 = 2 * (P_{sell})$ $Q_2 = 2 * (100 - \frac{1}{3}T)$
$Q_2 = 200 - \frac{2}{3}T$

(f) Calculate the tax revenue in terms of the tax.

Tax revenue is the tax (T) times the quantity Q_2

Tax Rev. $= T * (200 - \frac{2}{3}T)$
Tax Rev. $= 200T - \frac{2}{3}T^2$

(g) Calculate the DWL in terms of the tax.

DWL is the triangle such that the height is the change in quantity and the base is the tax (T).

$DWL = \frac{1}{2} * tax * (Q_1 - Q_2)$
$DWL = \frac{1}{2} * T * (200 - (200 - \frac{2}{3}T)$
$DWL = \frac{1}{2} * T * (\frac{2}{3}T)$
$DWL = \frac{1}{3}T^2$